Investigation on a Three-Stage Stirling-Type Pulse Tube Cryocooler for Cooling the Low-T_c SQUID

Zhiqian Gao, Haizheng Dang, Dingli Bao, and Yibo Zhao

***Abstract*—This paper presents the design and performance improvement of a three-stage Stirling-type pulse tube cryocooler (SPTC) aimed to achieve effective cooling at 4.2 K for the low-T_c superconducting quantum interference device (SQUID). The system integration of the three-stage SPTC and the low-T_c SQUID is described, and the cryocooler is arranged with the thermally coupled configuration. A developed electrical circuit analogy model is proposed to analyze the internal operating mechanisms of the three-stage SPTC. In addition, a three-stage thermally coupled SPTC is simulated and fabricated. The performance of each stage is tested individually. The first-stage SPTC can achieve 15.67 W at 80 K. A no-load cooling temperature of 17.4 K can be obtained by the two-stage SPTC with a total input acoustic power of 424.0 W. Under the precooling of the first two stages of SPTC, the simulated results indicate that the third-stage SPTC can acquire 6.35 mW at 4.2 K when the input acoustic power is 49.5 W, and the displacement of the cold end turns out to be 1.45 μm from peak to peak.**

***Index Terms*—Low-T_c SQUID, three-stage Stirling-type pulse tube cryocooler, design and performance improvement, system integration.**

I. Introduction

As THE most sensitive detector for magnetic field detection, the superconducting quantum interference device (SQUID) plays an important role in many kinds of weak magnetic detections, such as medical science, geomagnetism, nondestructive examination, etc. Especially for the low–T_c SQUID operating in a much lower temperature range of 4–9 K, it can achieve higher magnetic sensitivity and spatial resolution than its high–T_c counterpart, and also has stricter demand for the low-vibration and low-noise background. Therefore, many conventional mechanical cooling approaches become unsuitable due to their unacceptable vibration output and electromagnetic interference (EMI). The pulse tube cryocooler (PTC) eliminates any moving mechanical component at the cold end and thus has evident advantages at the cold end in terms of both low-vibration output and significantly reduced EMI levels. In addition, a flexible cooling temperature range can be realized by the PTC compared to the liquid helium cryostat. The Stirling-type PTC (SPTC) is driven by the linear compressor which features compact structure, low-vibration, high reliability and long operation time and thus become a desirable candidate for cooling the SQUID.

In order to obtain cooling capacities below 10 K, the SPTC with multi-stage arrangement is commonly used. Some of them were designed for space missions [1]–[3], which featured compact structure, but the cooling capacities were too tiny for cooling the low–T_c SQUID. Other aimed at the applications on ground [4], [5], and thus the structure could be somehow loose but the cooling powers became larger.

This paper presents the design and performance improvement of a three-stage SPTC aimed to achieve effective cooling at 4.2 K for the low–T_c SQUID. The system integration of the three-stage SPTC and the low–T_c SQUID are described and discussed. A developed electrical circuit analogy (ECA) model is proposed to analyze the internal operating mechanisms of the three-stage SPTC. In addition, a three-stage thermally-coupled SPTC is simulated and fabricated. The performance of each stage is tested individually, and the results are discussed in details.

II. Design of the Integration System

Fig. 1 shows the schematic diagram of the integration system for cooling the low–T_c SQUID. The three-stage SPTC is arranged with the thermally-coupled configuration, and driven by a Stirling-type helium compressor (1), which uses clearance seal and flexure springs, thus features a long operation life. A connecting tube (2) is adopted to eliminate the vibration transmitted from the compressor, its length and diameter are well optimized to reduce the irreversible losses.

The ambient heat exchangers of all the three stages of SPTC are mounted on the hot end exchanger (3), which is cooled to 290 K by a circulating water cooling system. For the first and second stage SPTCs, the cold fingers are coaxial with pulse tubes inside the regenerators, and thus have the advantage of compactness. But for the third stage, the pulse tube (18) and the reservoir (19) are precooled by the second stage cold heat exchanger (12), therefore, the U-type configuration is adopted instead of the coaxial one. Inertance tubes and reservoirs are

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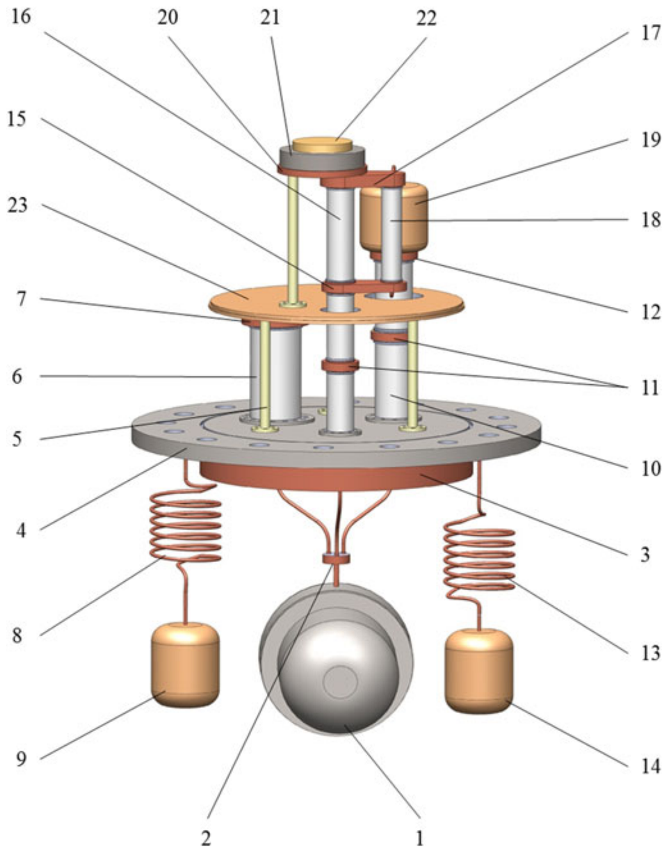

Fig. 1. Schematic diagram of the integration system for cooling the low-Tc SQUID: 1) compressor; 2) connecting tube; 3) hot heat exchanger; 4) hot end flange; 5) support frame; 6) first stage regenerator; 7) first stage cold heat exchanger; 8) first stage inertance tube; 9) first stage reservoir; 10) second stage regenerator; 11) first stage thermal link; 12) second stage cold heat exchanger; 13) second stage inertance tube; 14) second stage reservoir; 15) second stage thermal link; 16) third stage regenerator; 17) third stage cold heat exchanger; 18) third stage pulse tube; 19) third stage reservoir; 20) copper disk; 21) alumina SQUID holder; 22) low-Tc SQUID; 23) radiation shield plate.

used as the phase shifters in all the three stages of SPTC, it should be noted that, both of them work at ambient temperature in the first two stages of SPTC, while the third stage reservoir (19) works at about 30 K, therefore, it is located on the top of the second stage cold heat exchanger (12). The third stage inertance tube is assembled inside the reservoir (19) to ease the complexity of the integration system.

The low–T_c SQUID (22) is attached to a SQUID holder (21), which is made of alumina in order to provide sufficient thermal contact between the SQUID and the cold finger of the cryocooler. Metal can't be used for this purpose due to Johnson noise and eddy current effects. A radiation shield (23) cooled by the second stage cold heat exchanger (12) is placed around the low–T_c SQUID to reduce the radiation loss. The design goals of the integration system are shown in Table I.

III. Electrical Circuit Analogy Model for the Three-Stage SPTC

The ECA model is an effective approach of systematically analyzing a regenerative cryocooler, in which the dynamic pressure and the volume flow rate are analogized to the voltage and the current, respectively. With the aid of the analogy, Swift [6] systematically defined the equivalent elements of the key components such as regenerator, pulse tube, inertance tube and reservoir, respectively. Tan and Dang [7] proposed an ECA model for the whole system of a single-stage SPTC, and then Dang *et al.* [8] further improved the model by considering the effects of heat exchangers and the compression space. However, both of the above two models were developed based on the single-stage SPTC working above about 50 K, in which the working fluid, helium, were regarded as the ideal gas. For the discussed three-stage SPTC model with the coldest stage working at or even below 10 K, the real gas effect should be considered. Therefore, a still further improved ECA model considering the effects of the real gas is proposed as follows.

TABLE I
Design Goals of the Integration System

Parameters	Design goals
cooling capacity	≥6 mW/4.2 K
input electrical power	≤ 800 W
total mass	≤ 50 kg
peak to peak displacement	≤ 1.5 μm
operation lifetime	≥ 3 y

In the developed ECA model, the dynamic pressure, the volume flow rate and the dynamic temperature are discussed with harmonic approximations, and the model is based on cross-sectional-averaged equations. Therefore, the momentum equation becomes [6]:

$$\Delta \boldsymbol{p_d} = -\frac{i\omega\rho_m \Delta x/A}{1-(1-i)\delta_v/2r_h}\dot{\boldsymbol{U}} \cong -\left(\frac{i\omega\rho_m\Delta x}{A}+R_v\right)\dot{\boldsymbol{U}} \tag{1}$$

In the pulse tube, the thermodynamic process can be regarded as adiabatic, thus its continuity equation can be expressed as [6]–[9]:

$$\boldsymbol{p_d} = -\frac{\gamma_m p_m}{i\omega A\Delta x}\Delta\dot{\boldsymbol{U}} + \frac{p_m \ln p_m}{\gamma_m}\left(\frac{\partial\gamma}{\partial p}\boldsymbol{p_d}+\frac{\partial\gamma}{\partial T}\boldsymbol{T_d}\right) \tag{2}$$

In the regenerator, the process is not a simple adiabatic one, therefore, a controlled source item $g\dot{\boldsymbol{U}}$ is added for correction as follows [6]–[9]:

$$\boldsymbol{p_d} = -\frac{\gamma_m p_m}{i\omega A\Delta x}(\Delta\dot{\boldsymbol{U}}+g\dot{\boldsymbol{U}}) + \frac{p_m \ln p_m}{\gamma_m}\left(\frac{\partial\gamma}{\partial p}\boldsymbol{p_d}+\frac{\partial\gamma}{\partial T}\boldsymbol{T_d}\right) \tag{3}$$

And if considering the isothermal processes in heat exchangers, inertance tubes and reservoirs, their continuity equation turns out to be [6]–[9]:

$$\boldsymbol{p_d} = -\frac{p_m}{i\omega A\Delta x}\Delta\dot{\boldsymbol{U}} + \frac{p_m}{C_{fm}}\left(\frac{\partial C_f}{\partial p}\boldsymbol{p_d}+\frac{\partial C_f}{\partial T}\boldsymbol{T_d}\right) \tag{4}$$

where $\boldsymbol{p}_d$, $\dot{\boldsymbol{U}}$, $\boldsymbol{T}_d$ are the dynamic pressure, the volume flow rate and the dynamic temperature, respectively, A is the cross-sectional area, R_v is the viscous resistance, δ_v is the viscous penetration depth, r_h is the hydraulic radius, γ is the specific heat ratio, C_f is the compressibility factor, g is the complex gain

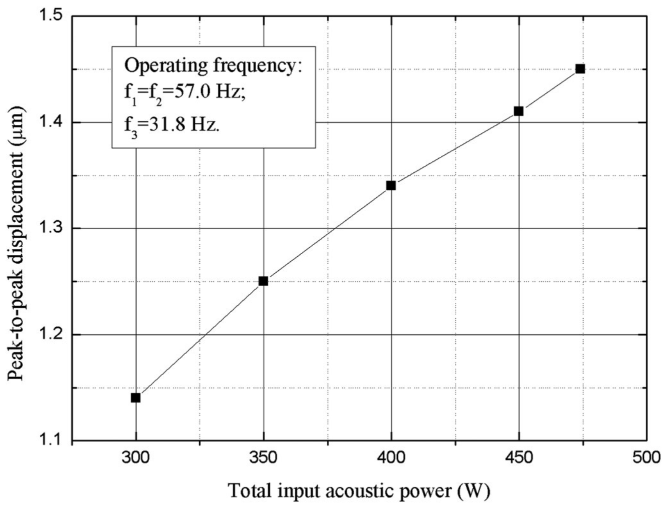

Fig. 2. Displacement of the three-stage cold heat exchanger.

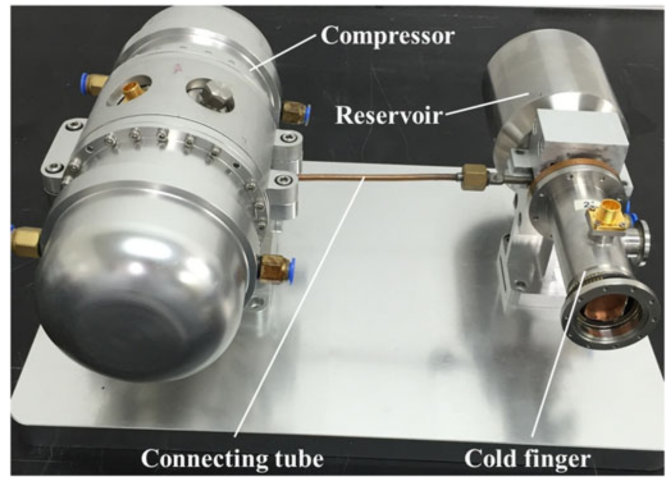

Fig. 3. Single-stage SPTC configuration.

factor for volume flow rate [6], and the subscript *m* in (1)–(4) represents the mean values.

Combing the above equations, the dynamic pressure and the volume flow rate in different components of the three-stage thermally-coupled SPTC can be obtained quantitatively. In addition, the irreversible losses in regenerators and heat exchangers can also be worked out according to the earlier work [9]. Therefore, the cooling capacities of each stage SPTC can be predicted definitely.

IV. Experiments and Discussions

A three-stage thermally-coupled SPTC aimed at 4.2 K is designed for cooling the low–T_c SQUID based on the above theoretical model. The first stage cold finger is fabricated and tested as a single-stage cryocooler separately, and then the second stage is added for a two-stage cryocooler. Finally, the third stage is designed on the basis of the first and second stages.

In order to evaluate the feasibility of the three-stage SPTC for cooling a low–T_c SQUID, we tested the peak-to-peak displacement of the third stage cold heat exchanger along the axial direction of the pulse tube. With the increasing input acoustic power, the displacement of the cold end increases monotonically as shown in Fig. 2. The largest displacement is 1.45 μm when the input acoustic power is 474 W, which turns out to be the optimal input acoustic power for the three-stage SPTC.

Compared with other mechanical cryocoolers operating at 4.2 K, the vibration generated by the three-stage SPTC is much smaller. For the G-M type PTC, it has a peak-to-peak displacement of about 50 μm at 1.4 Hz [10]. For the G-M cryocooler and the Stirling cryocooler, the vibrations are one order of magnitude larger than that of G-M type PTC due to the moving displacer at the cold end [11]. The Joule-Thomson cryocooler can't reach 4.2 K directly due to the thermodynamic properties of the working fluid, helium, unless it is precooled by one of the above-mentioned cryocooler. Therefore, the developing multi-stage SPTC turns out to be one of the best choices for cooling the low–T_c SQUID.

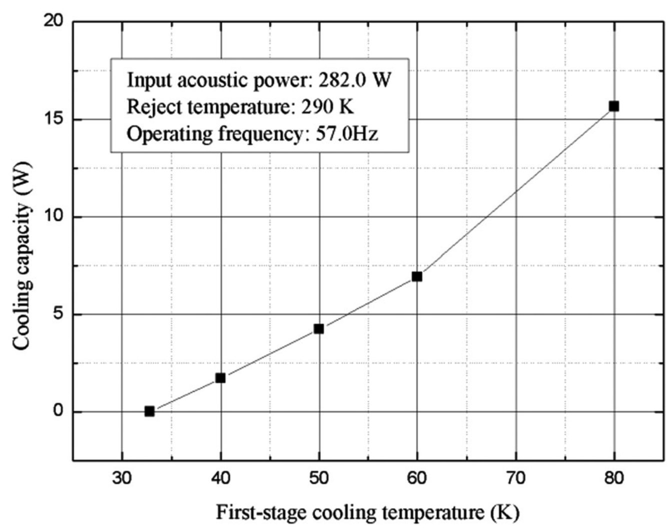

Fig. 4. Experimental performance of the single-stage SPTC.

A. The First Stage

The first stage SPTC is designed based on the single-stage SPTC operating at 30-60 K developed in the same laboratory [12]–[15], the configuration is shown in Fig. 3. A dual-opposed compressor is adopted with an input acoustic power of 282.0 W, and the cold finger is arranged with the coaxial configuration to realize the advantage of compactness for the integration system. #400 stainless steel screens are used as the regenerator matrix. Two inertance tubes are adopted as the phase shifter.

The performance of the single-stage SPTC is shown in Fig. 4. When the operating frequency is 57.0 Hz, the charging pressure is 3.3 MPa, and the hot end is cooled to 290 K, a no-load cooling temperature of 32.8 K can be obtained. At the cooling temperature of 80 K, the cooling capacity turns out to be 15.67 W.

B. The Second Stage

The second stage cold finger is added to the above-mentioned single-stage SPTC, and tested as a two-stage SPTC, the

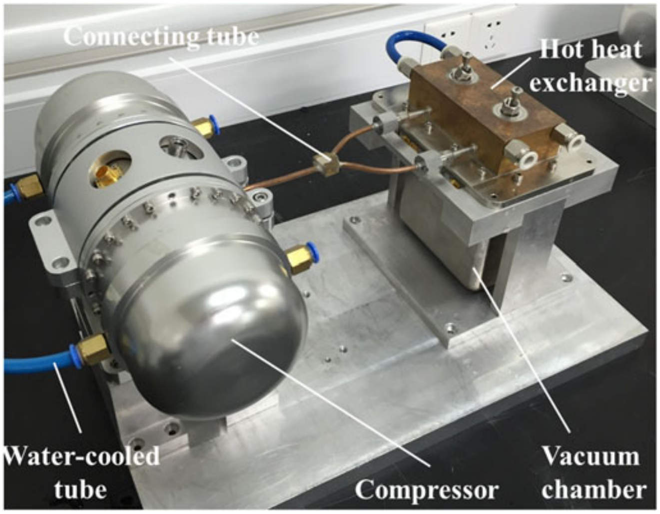

Fig. 5. Two-stage SPTC configuration.

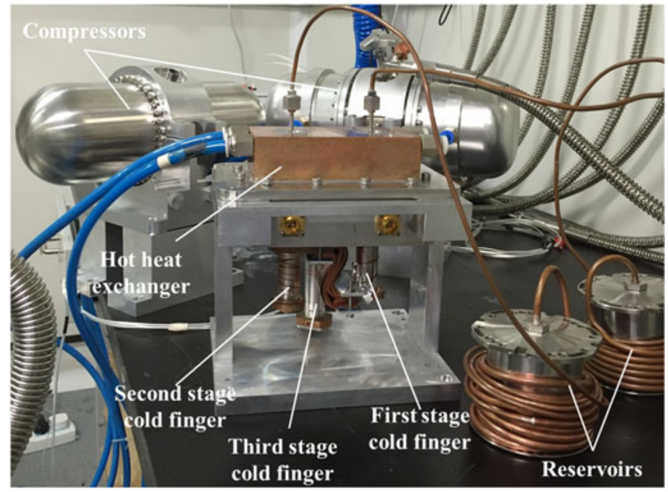

Fig. 7. Experimental setup of the three-stage SPTC.

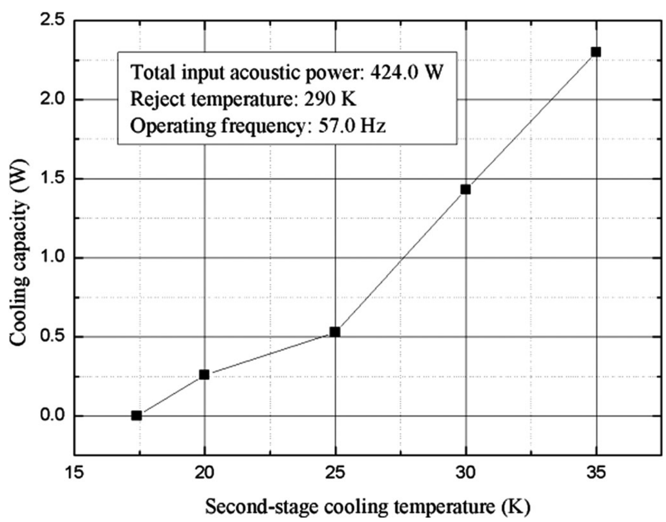

Fig. 6. Experimental performance of the two-stage SPTC.

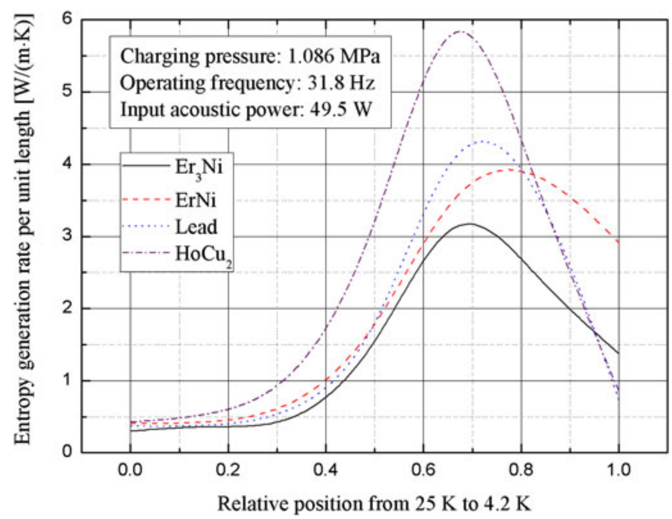

Fig. 8. Entropy generation rates caused by different matrix.

structure is shown in Fig. 5. The coaxial configuration is employed again to the cold finger. A thermal link made of copper strap is designed to connect the first stage cold heat exchanger and the second stage regenerator. The thermal link is flexible to avoid the stretching force caused by the contraction under low temperature. Two options have been tried for the second stage regenerator matrix: 1) a layered structure of stainless steel screens and Er_3Ni, with stainless steel screens from the hot end to the thermal link, then Er_3Ni for the thermal link to the cold end, and 2) a completely packing for only Er_3Ni. The experimental results indicate the second option is a little better than the first one.

The performance of the two-stage SPTC is shown in Fig. 6. When no cooling capacities are extracted from the first stage SPTC, the precooling temperature becomes 64.4 K, and the no-load cooling temperature of the second stage SPTC turns out to be 17.4 K with a total input acoustic power of 424.0 W. The reject temperature is 290 K, and the operating frequency is 57.0 Hz.

C. The Third Stage

The third stage SPTC is designed to reach 4.2 K for cooling the low–T_c SQUID based on the single-stage and two-stage SPTCs. The experimental setup is shown in Fig. 7. Unfortunately, several components are still under manufacturing. Therefore, only simulated results will be reported here.

A layered structure is used as the regenerator matrix, with #400 stainless steel screens from 290 K to 80 K, #635 stainless steel screens from 80 K to 30 K, and for the last section from 30 K to 4.2 K, different matrix is simulated and compared with each other according to the ECA model. The simulated results shown in Fig. 8 indicate that $HoCu_2$ may cause the largest irreversible losses, followed by Lead, ErNi and Er_3Ni. Therefore, Er_3Ni becomes the best option.

The operating frequency and the charging pressure of the third-stage SPTC are optimized as well. The optimal values turn out to be 31.8 Hz and 1.086 MPa, respectively.

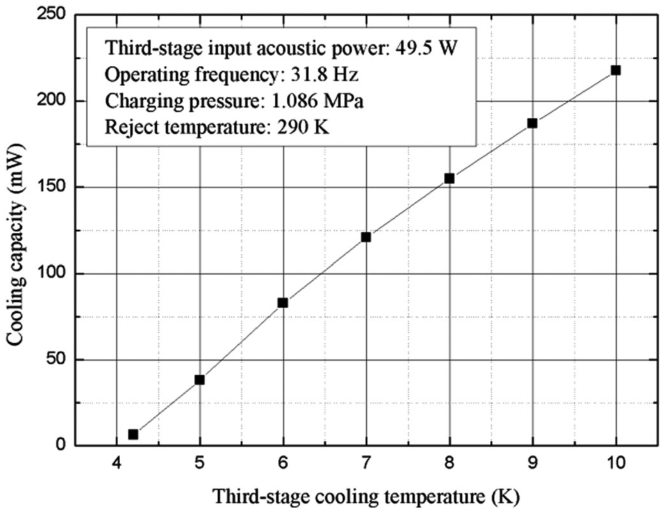

Fig. 9. Simulated performance of the third-stage SPTC.

Under these conditions, a cooling capacity of 6.35 mW at 4.2 K can be obtained by the third-stage SPTC, when the third-stage input acoustic power is 49.5 W, and the precooling capacities provided by the first two stages of SPTC are 2.52 W at 80 K and 1.43 W at 30 K, respectively. Meanwhile, the cooling capacity would increase to 186.8 mW, when the cooling temperature becomes 9 K. The detailed performance is shown in Fig. 9.

V. Conclusion

The low–T_c SQUID needs to work in the temperature range of 4–9 K, and has a demanding requirement to the vibration level. Therefore, a three-stage thermally-coupled SPTC is introduced in this paper, and the system integration is described in details. In addition, an improved ECA model is proposed to analyze the internal flows of the three-stage SPTC by considering the real gas effects of helium. A three-stage thermally-coupled SPTC is designed, and each stage SPTC is tested individually. The experimental results indicate that the first stage SPTC can achieve 15.67 W at 80 K when the input acoustic power is 282.0 W. The second stage SPTC can reach a no-load cooling temperature of 17.4 K under the precooling of the first stage. For the third stage, the simulated results show it can acquire 6.35 mW at 4.2 K with the precooling of the first two stages of SPTC, when the input acoustic power of the third stage is 49.5 W. Under these conditions, the peak to peak displacement of the third stage cold heat exchanger turns out to be 1.45 μm.

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