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Investigation and optimization of the continuous and discrete heat exchangers in the mK dilution refrigerator for cooling superconducting quantum chips

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Abstract. The recent decade has witnessed the rapid development of superconducting quantum computing in which its chips require to operate below 100 mK. The dilution refrigerator, which can operate continuously and stably and features the merits of low vibration and electromagnetic Interference, has become the enabling cooling technology for this purpose. There are two types of heat exchangers in the dilution refrigerator based on the effect of Kapitza resistance, namely the continuous and discrete heat exchangers. Their flow and heat transfer characteristics are crucial to the refrigerator's performance. In this paper, a numerical model of the heat exchangers in the dilution refrigerator is established, based on which the effects of dimensions, the Kapitza thermal resistance, and viscous heating on the cooling performance are investigated. The optimal parameters are given, and the cooling capacity of the dilution unit can be improved from 19 μW to 25.2 μW at 20 mK theoretically, which would provide a better temperature environment for quantum computers requiring more qubits.

1. Introduction

The recent decade has witnessed the rapid development of quantum information technology, and the demand for dilution refrigerators (DRs) has also increased. They provide the millikelvin environment for quantum chips and sufficient cooling capacity to accommodate more qubits, becoming an irreplaceable key technology in quantum computing. There are two types of heat exchangers in the DR below 1 K, including the continuous heat exchanger (CHEX) and the discrete heat exchangers (DHEXs). Both of them play decisive roles in the performance characteristics of the DR because heat transfer at ultra-low temperatures is quite complicated and determines the cooling performance of the DR. Thus, they require accurate design. However, specific low-temperature effects below 1 K, such as the variable fluid properties of the helium mixture, axial heat conduction, and viscous heating, make conventional modeling approaches inapplicable.

Some previous theoretical analyses of heat exchangers in the DR have been conducted. Siegwaeth et al. [1] analyzed the role of axial thermal conductivity in continuous and discrete heat exchangers. Still, assuming no temperature gradient through the solid component leads to a selfcontradiction. Staas et al. [2] proposed the surface efficiency of DHEXs and discussed how to avoid the adverse effects of gravity instability, thermal conductivity, and viscosity. Frossati et al. [3] investigated the CHEX model and DR's performance under appropriate boundary conditions. Takano [4] evaluated the cooling capacity of DR with a perfect CHEX. However, axial heat conduction and viscous heating were excluded from both studies. Oda et al. [5] designed the heat exchanger structure according to the overall performance requirements and adopted the DHEXs with a bypass to reduce the flow resistance. Mueller et al. [6] developed a numerical model of a cold-cycle dilution refrigerator but neglected the viscous heating.

Based on the above studies, this paper mainly focuses on optimizing two types of heat exchangers in terms of structure and performance. It establishes a numerical model of the heat exchangers, based on which the effects of dimensions, Kapitza thermal resistance, and viscous heating are investigated, and the calculation results are discussed.

2. Theoretical analysis and numerical modeling

2.1 Physical model

The working fluid ^3He-^4He mixture separates into the concentrated phase (*c*-phase) and dilute phase (*d*-phase) of ^3He at 0.86 K in the mixing chamber. When the dilute phase flows into the still driven by osmotic pressure, the ^3He in the *c*-phase passes through the phase interface to the d-phase. This entropy-increasing process is the basic cooling principle of the DR. Two types of heat exchangers below 1 K are shown in Figure 1.

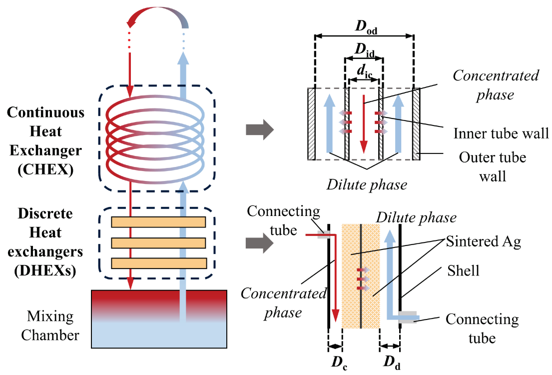

Figure 1. Schematic diagram of CHEX and DHEXs below 1 K

The continuous heat exchanger is similar to the tube-in-tube type heat exchanger. The inner and outer tubes are made of copper-nickel alloy (Cu-Ni) due to its moderate thermal conductivity at ultra-low temperatures. The *c*-phase flows in the inner tube, while the *d*-phase is in the annular space between the inner and outer tube. As for the discrete heat exchangers, namely the sintered-Ag heat exchangers, when the temperature goes to about 0.1 K, the Kapitza resistance will become quite large and the main factor affecting the heat transfer. Therefore, it is necessary to adopt the DHEXs in the DR so that the heat transfer deterioration can be remedied by sintered silver blocks that maximize the heat transfer area in the limited space. Usually, there are 4 to 6 stages of DHEX in the system, and each stage is composed of a thin Cu-Ni plate, two sintered-Ag blocks, and two stainless steel shields. The liquid *c*-phase enters the upper flow channel while the *d*-phase liquid mixture is in the lower one.

2.2 Theoretical analysis

To simplify the following analysis, the following assumptions are taken into consideration:

(1) All the fluids are steady, the flow rate remains constant, and the temperature distribution is uniform.

(2) The sintered Ag blocks have homogeneous and isotropic microscopic structures and are in ideal thermal contact with the CuNi plate.

(3) The temperature and velocity gradients in the radial direction are neglected.

According to the conservation of energy, the coupled heat transfer governing equation of the heat exchangers below 1K is:

$$X\frac{d}{dx}\left(\frac{dT_{\mathrm{i}}}{dx}\right)+\dot{n}V_{\mathrm{i}}\left|\frac{dp_{\mathrm{i}}}{dx}\right|+Z\left(T_{\mathrm{c}}^{4}-T_{\mathrm{d}}^{4}\right)=\dot{n}C_{\mathrm{i}}\frac{dT_{\mathrm{i}}}{dx} \tag{1}$$

The three terms on the left represent axial heat conduction, viscous heating, and lateral interface heat transfer. In the second one, the pressure drop per unit length of the fluid is:

$$\left|\frac{dp_{\mathrm{i}}}{dx}\right|=\frac{128\eta_{\mathrm{i}}\dot{n}V_{\mathrm{i}}}{\pi D_{\mathrm{i}}^{4}} \tag{2}$$

where D_{i} is the generalized equivalent diameter.

Thus, the governing equation (1) can be summarized as:

$$X\frac{d}{dx}\left(\frac{dT_{\mathrm{i}}}{dx}\right)+Y\frac{128\dot{n}^{2}}{\pi}+Z\left(T_{\mathrm{c}}^{4}-T_{\mathrm{d}}^{4}\right)=\dot{n}C_{\mathrm{i}}\frac{dT_{\mathrm{i}}}{dx} \tag{3}$$

The coefficients *X*, *Y*, and *Z* of the *c*-phase (*i*=c) and *d*-phase (*i*=d)in the continuous and discrete heat exchangers are listed in Table 1. In the following expressions, *k* is the intrinsic thermal conductivity, η is the viscosity of the working fluid, $\dot{n}$ is the molar flow rate, and *V* is the molar volume, R_{k} is the Kaptiza resistance:

$$\eta_{c}=2.2T^{-2}+26.3T^{-\frac{1}{3}}\ \mu\mathrm{P} \tag{4}$$

$$\eta_{d}=0.5T^{-2}\ \mu\mathrm{P} \tag{5}$$

$$V_{\mathrm{c}}=36.84\ \mathrm{cm^{3}/mol} \tag{6}$$

$$V_{\mathrm{d}}=27.58(1+0.286x)\ \mathrm{cm^{3}/mol} \tag{7}$$

In addition, d_{ic}, D_{ic}, and D_{id} are the inner diameter of the inner tube, the outer diameter of the inner tube, and the inner diameter of the outer tube in the CHEX, respectively. D_{c} and D_{d} are the equivalent diameters of the channels on concentrated and dilute sides in the DHEXs. φ is the porosity of the sintered Ag block, while γ is the surface-volume ratio.

According to the enthalpy balance in the mixing chamber, the cooling capacity of the DR is:

$$Q_{\mathrm{mc}}=\dot{n}(H_{\mathrm{d}}(x_{s},T_{\mathrm{mc}})-H_{3}(T_{\mathrm{in}})) \tag{8}$$

where T_{mc} is the minimum temperature of DR, T_{in} is the inlet temperature of the mixing chamber, and x_{s} is the concentration of the saturated solution.

2.3 Numerical model

Based on the above theoretical analyses, a finite difference model is set up to solve the equation to get the thermal performance. The heat exchangers are regarded as a one-dimensional model and divided into N segments (N+1 points) along the length. The mesh is shown in Figure 2, and the calculation is conducted by MATLAB.

The boundary conditions are listed as follows:

$$T_{\mathrm{c,N+1}}=T_{\mathrm{s}}=0.7\ \mathrm{K} \tag{9}$$

$$T_{\mathrm{d,1}}=T_{\mathrm{mc}} \tag{10}$$

It is assumed that there is no heat loss in the tube between each component below 1 K; that is to say, the inlet temperature of the CHEX is equal to the outlet temperature of the still, and the inlet temperature of the DHEXs is equal to the minimum temperature of the mixing chamber. The outlet temperature of the CHEX is also the inlet temperature of the first stage DHEX, represented by $T_{c,\,mid}$ in the following content. The molar flow rate is 800 µmol/s.

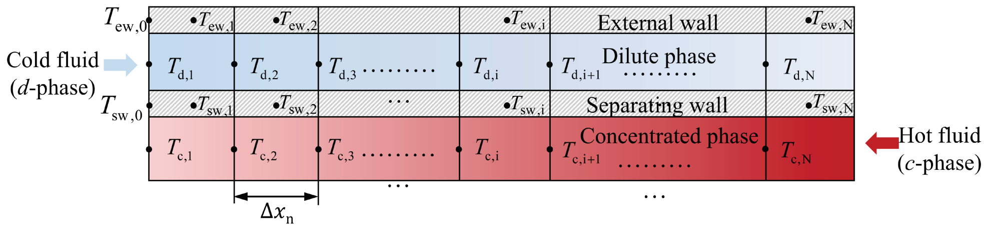

Figure 2. Gridded control volume of the heat exchanger

Table 1. Coefficients in equation(3) of *c*-phase and *d*-phase in the CHEX and DHEXs

	CHEX (*c*-phase)	CHEX (*d*-phase)	DHEX (*c*-phase)	DHEX (*d*-phase)
X	$\frac{\pi d_{ic}^2}{4} k_c$	$\frac{\pi (D_{od}^2 - D_{id}^2)}{4} k_d$	$\left(\varphi A_{sintered\text{-}Ag} + \frac{\pi D_c^2}{4}\right) k_c$	$\left(\varphi A_{sintered\text{-}Ag} + \frac{\pi D_d^2}{4}\right) k_d$
Y	$\frac{\eta_c V_c^2}{d_{ic}^4}$	$\frac{\eta_d V_d^2}{(D_{od} - D_{id})^2 (D_{od}^2 - D_{id}^2)}$	$\frac{\eta_c V_c^2}{D_c^4}$	$\frac{\eta_d V_d^2}{D_d^4}$
Z	$\frac{A_{ic}}{4 R_{k,c}}$	$\frac{A_{od}}{4 R_{k,d}}$	$\frac{\gamma A_{sin\,tered-Ag}}{4 R_{k,c}}$	$\frac{\gamma A_{sin\,tered-Ag}}{4 R_{k,d}}$

2.4 Thermal properties

In two types of heat exchangers, both the *c*-phase and the *d*-phase are liquid fluids. The *c*-phase can be regarded as pure ^3He, while the *d*-phase is a ^3He-^4He liquid mixture. The thermal properties of pure ^3He are obtained from He3Pak [7], and those of the mixture have been discussed and calculated in our team's previous paper [8].

3. Discussions

3.1 Effects of dimensions

The dimensions, such as the length and diameters of the CHEX, determine the heat exchanger area and are critical factors in the heat transfer performance. The performance of the CHEX is deduced with the conditions of d_{ic}=0.6mm, D_{id}=1.0mm, D_{od}=4mm, and the length from 0.2 to 3 m.

As is shown in Figure 3, with the increasing length, there is an initial sharp fall followed by a slow fall in the outlet temperature of the concentrated side due to the increase of heat exchanger area. Meanwhile, that of the dilute side is the contrary and the cooling power increases. When the length increases over 2.6 m, the outlet temperature of both sides remains unchanged, but the entropy generation becomes higher, leading to irreversible loss. Thus, with a length of 2.6 m, the optimal heat exchange area is about 82 mm^2, and the cooling power is 25.2 µW.

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With the constant heat exchanger area, the performance with different diameters is also conducted with d_{ic} from 0.5 to 0.7 mm and D_{od} from 3 to 5 mm. As shown in Figure 4, the outlet temperature of both sides is almost unchanged, but the pressure drop and viscous heating decrease with the increase of d_{ic}. However, it is accompanied by an increase in irreversible losses when d_{ic} is over 0.6mm due to the liquid axial heat conduction. As for D_{od}, only the pressure drop on the dilute side is reduced, and viscous heating is weakened, which is helpful for the heat exchanger. Considering the pressure drop and the entropy generation, the optimal dimensions of CHEX are L=2.6 m, d_{ic}=0.6 mm, and D_{od}=4 mm.

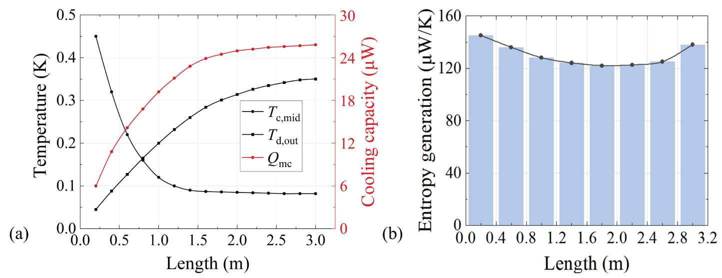

Figure 3·. Effects of the length of CHEX on outlet temperature, cooling capacity, and entropy

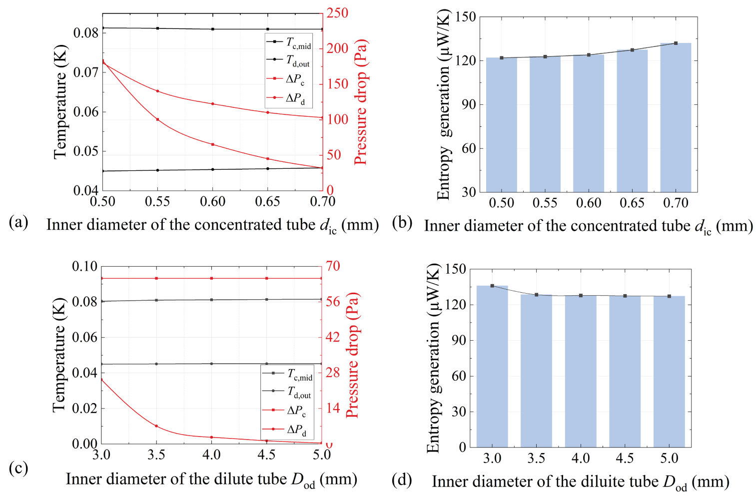

Figure 4. Effects of d_{ic} and D_{od} on outlet temperature, pressure drop, and entropy

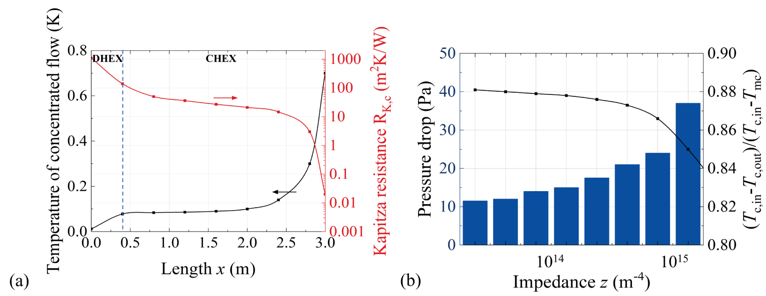

Figure 5. (a) Kapitza thermal resistance and temperature over the whole length of the heat exchanger; (b) The relationship between the impedance, pressure drop, and heat exchange performance

3.2 Effects of Kapitza thermal resistance

Kapitza resistance is the decisive factor in the HEXs of DR, and its distribution is shown in Figure 5(a). At the entrance of CHEX, the concentrated phase is cooled from 0.7 K to 0.1 K with a length of 1 m, and the Kapitza resistance is small above 0.1 K. As the temperature decreases, it becomes enormous and is the main factor influencing the heat exchange in the DHEX. Thus, the DHEXs use sintered Ag blocks with ultra-large specific surface areas to achieve further cooling.

3.3 Effects of the viscous heating

Additionally, viscous heating, which depends on the impedance of the heat exchangers, plays a vital role in the cooling performance, especially in the DHEXs below 0.1 K. The impedance is related to the geometric parameters of flow channels. The difference of the heat exchanger inlet and outlet temperature divided by the difference of the inlet temperature and mixing chamber temperature can be utilized to estimate the performance of HEXs. As shown in Figure 5(b), when the impedance increases from 2×10^{14} m^{-4} to 1×10^{15} m^{-4}, the ratio drops by 15%, and the pressure drop increases by 146%. Therefore, to avoid the sharp performance degradation of heat exchangers, the impedance should be limited to 2×10^{14} m^{-4}.

4. Conclusions

The rapid development of quantum computing has put forward increasing demands for the dilution refrigerator in which the continuous and discrete heat exchangers play crucial roles in determining the cooling performance. In this paper, a numerical model is established based on the finite difference model and the governing equation of heat exchangers. Utilizing the model, the effects of various factors are investigated, and the performance of heat exchangers is optimized. For the CHEX, the optimal dimensions are L=2.6 m, d_{ic}=0.6 mm, and D_{od}=4 mm, at which the outlet temperature of the concentrated phase decreases by 15.8% (from 76 mK to 64 mK). In addition, the Kapitza resistance becomes the main factor influencing the heat exchange characteristics of DHEX. The impedance should be confined to 2×10^{14} m^{-4} to reduce the degeneration caused by viscous heating. Compared to the dilution refrigerator with original heat exchangers, the cooling capacity of it with the optimized heat exchangers can be improved from 19 μW to 25.2 μW at 20 mK theoretically, which allows it to accommodate more qubits in the quantum computers.

Acknowledgments

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